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MISCELLANEOUS.

173. Proposed by G. B. M. ZERR, A. M., Ph. D., Philadelphia, Pa.

If n is odd, prove the following: $\pm 1 = [(-1)^{1/n} + (-1)^{-(1/n)}] [(-1)^{2/n} + (-1)^{-(2/n)}] [(-1)^{3/n} + (-1)^{-(3/n)}] \dots [(-1)^{(n-1)/2n} + (-1)^{-(n-1)/2n}] \pm \sqrt[n]{n} (-1)^{(n-1)/4} = [(-1)^{1/n} - (-1)^{-(1/n)}] [-(-1)^{2/n} - (-1)^{-(2/n)}] [(-1)^{3/n} - (-1)^{-(3/n)}] \dots [(-1)^{(n-1)/2n} - (-1)^{-(n-1)/2n}]$.

Solution by the PROPOSER.

Delete the factor $(-1)^{(n-1)/4}$ from the first member of the second expression. To resolve into factors the expression $x^n - 1 = 0$. When n is odd,

$$x^n = 1 = \cos 2m\pi \pm i \sin 2m\pi = (-1)^{\pm 2m},$$

$$\text{since } \cos 2m\pi = \frac{1}{2} [(-1)^{2m} + (-1)^{-2m}], \quad \sin 2m\pi = \frac{1}{2i} [(-1)^{2m} - (-1)^{-2m}].$$

Giving m the values 0, 1, 2, 3, etc.,

$$x = (-1)^{\pm 0}, (-1)^{\pm (2/n)}, (-1)^{\pm (4/n)}, \dots, (-1)^{\pm (n-1)/n}.$$

\therefore The roots are 1, $(-1)^{\pm (2/n)}$, $(-1)^{\pm (4/n)}$, ..., $(-1)^{\pm (n-1)/n}$, and the factors are $(x-1)$, $[x - (-1)^{2/n}] [x - (-1)^{-(2/n)}]$, $[x - (-1)^{4/n}] [x - (-1)^{-(4/n)}]$, ..., $[x - (-1)^{(n-1)/n}] [x - (-1)^{-(n-1)/n}]$.

$$\therefore x^n - 1 = (x-1) [x^2 - x(-1)^{2/n} - x(-1)^{-(2/n)} + 1] [x^2 - x(-1)^{4/n} - x(-1)^{-(4/n)} + 1], \dots, [x^2 - x(-1)^{(n-1)/n} - x(-1)^{-(n-1)/n} + 1].$$

If $x = -1$, $(x^n - 1)/(x - 1) = 1$.

$$\therefore 1 = [2 + (-1)^{2/n} + (-1)^{-(2/n)}] [2 + (-1)^{4/n} + (-1)^{-(4/n)}],$$

$$\dots, [2 + (-1)^{(n-1)/n} + (-1)^{-(n-1)/n}],$$

$$= [(-1)^{1/n} + (-1)^{-(1/n)}]^2 [(-1)^{2/n} + (-1)^{-(2/n)}]^2,$$

$$\dots, [(-1)^{(n-1)/2n} + (-1)^{-(n-1)/2n}]^2.$$

$$\therefore \pm 1 = [(-1)^{1/n} + (-1)^{-(1/n)}] [(-1)^{2/n} + (-1)^{-(2/n)}] [(-1)^{3/n} + (-1)^{-(3/n)}] \dots [(-1)^{(n-1)/2n} + (-1)^{-(n-1)/2n}].$$

If $x = 1$, $(x^n - 1)/(x - 1) = x^{n-1} + x^{n-2} + \dots + x + 1 = n$.

$$\therefore n = [(-1)^{1/n} - (-1)^{-(1/n)}]^2 [(-1)^{2/n} - (-1)^{-(2/n)}]^2,$$

$$\dots, [(-1)^{(n-1)/2n} - (-1)^{-(n-1)/2n}]^2.$$

$$\therefore \pm \sqrt[n]{n} = [(-1)^{1/n} - (-1)^{-(1/n)}] [(-1)^{2/n} - (-1)^{-(2/n)}] [(-1)^{3/n} - (-1)^{-(3/n)}] \dots [(-1)^{(n-1)/2n} - (-1)^{-(n-1)/2n}].$$

175. Proposed by PROFESSOR R. D. CARMICHAEL, Anniston, Ala.

If x and z are connected by the relation $z = zf(x) + x\phi(z)$, find the value of $f(z)$ in the form of a power series in x with constant coefficients. In particular, give such a value of z when $z = z \sin x + x \cos z$.